

# Back-action cancellation in interferometers by quantum locking

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We show that back-action noise in interferometric measurements such as gravitational-waves detectors can be completely suppressed by a local control of mirrors motion. An optomechanical sensor with an optimized measurement strategy is used to monitor mirror displacements. A feedback loop then eliminates radiation-pressure effects without adding noise. This very efficient technique leads to an increased sensitivity for the interferometric measurement, which becomes only limited by phase noise. Back-action cancellation is furthermore insensitive to losses in the interferometer.

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Sensitivity in interferometric measurements such as gravitational-waves detectors [1, 2] is ultimately limited by quantum noise of light. Phase fluctuations introduce noise in the measurement whereas radiation pressure of light induces unwanted mirrors displacements. Both lead to a quantum limit and potential applications of squeezed states to overcome this limit have motivated a large number of works in quantum optics [3, 4, 5, 6].

It has recently been proposed to enhance the sensitivity in interferometric measurement by active control of mirrors displacements [7]. Active control can reduce classical noise such as thermal noise in cold-damped mechanical systems [8, 9, 10, 11] and may in principle be used to reduce noise in a quantum regime [12, 13, 14].

The scheme proposed in [7] is based on a local control of each mirror of the interferometer. An optomechanical sensor made of a high-finesse cavity monitors the mirror motion. A feedback loop then locks the mirror at the quantum level, with respect to the position of the other mirror of the sensor cavity. The sensor sensitivity is transferred to the interferometric measurement, resulting in a reduction of back-action noise due to radiation pressure in the interferometer.

In this paper we show that a similar technique can be used to completely suppress back-action noise. An optimized measurement strategy for the optomechanical sensor allows one to freeze the mirror in an absolute way, leading to a complete elimination of radiation-pressure noise. We furthermore show that this behavior is insensitive to the characteristics of the interferometer. In contrast to injection of squeezed states [6], back-action cancellation is still obtained in presence of losses in the interferometer.

The basic setup is shown in Fig. 1. We focus on the active control of one mirror of the interferometer (mirror  $m$  in Fig. 1). The interferometric measurement is schematized as the measurement of a length variation  $X_{\text{sig}}$  of a Fabry-Perot cavity which can be considered as one of the two arms of a gravitational-wave detector (left part of Fig. 1). Motion of mirror  $m$  relative to the position of a reference mirror  $r$  is measured by an optomechanical sensor made of both mirrors. The intensity of the field reflected by this high-finesse cavity is measured after be-

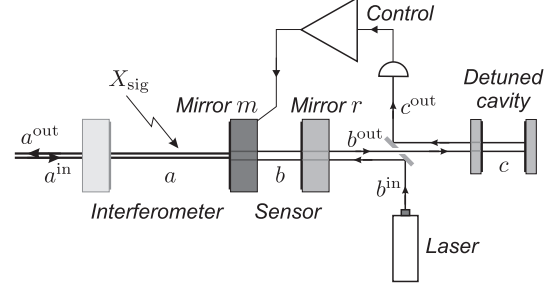


FIG. 1: Scheme of the system studied in the paper. A Fabry-Perot cavity and a light field  $a$  are used to measure a variation  $X_{\text{sig}}$  of the cavity length. Mirror  $m$  is actively controlled by a feedback loop via the position measurement delivered by an optomechanical sensor (cavity made of mirrors  $m$  and  $r$  with a light field  $b$ , followed by a detuned cavity).

ing phase-shifted by a detuned cavity. The result of the measurement is fed back to the mirror in order to control its displacements.

Fields  $a$  and  $b$  in both the interferometer and sensor are described by quantum annihilation operators  $a[\Omega]$ ,  $b[\Omega]$  at frequency  $\Omega$ , whereas mean fields are characterized by complex amplitudes  $\alpha$ ,  $\beta$  normalized in such a way that  $|\alpha|^2$  and  $|\beta|^2$  correspond to photon fluxes [15]. We define in a usual way the quadrature  $a_\theta$  of field  $a$  as,

$$a_\theta[\Omega] = e^{-i\theta} a[\Omega] + e^{i\theta} a^\dagger[\Omega]. \quad (1)$$

When the mean field  $\alpha$  is real, intensity and phase quadratures respectively correspond to the quadrature  $a_0$  aligned with the mean field and to the orthogonal quadrature  $a_{\frac{\pi}{2}}$ .

For a lossless and resonant single-ended cavity, the incident, intracavity, and reflected mean fields can be taken real. Assuming the frequency  $\Omega$  of interest smaller than the cavity bandwidth, the input-output relations for the interferometer cavity are given by [14],

$$\gamma_a a = \sqrt{2\gamma_a} a^{\text{in}} + 2ik_0 \alpha (X_m + X_{\text{sig}}), \quad (2)$$

$$a^{\text{out}} = -a^{\text{in}} + \sqrt{2\gamma_a} a, \quad (3)$$

where  $\gamma_a$  is the damping rate of the cavity,  $k_0$  the field wavevector and  $X_m$  the displacement of mirror  $m$ . The

intensity quadrature is left unchanged by the cavity ( $a_0^{\text{out}} = a_0^{\text{in}}$ ) whereas the input-output phase-shift is proportional to the cavity length variation,

$$a_{\frac{\pi}{2}}^{\text{out}} = a_{\frac{\pi}{2}}^{\text{in}} + 2\xi_a (X_m + X_{\text{sig}}). \quad (4)$$

The optomechanical coupling parameter  $\xi_a$  is related to the intracavity mean field amplitude  $\alpha$  and to the cavity finesse  $\mathcal{F}_a = \pi/\gamma_a$ ,

$$\xi_a = 2k_0\alpha\sqrt{2\mathcal{F}_a/\pi}. \quad (5)$$

Measurement of the phase of the reflected field provides an estimator  $\hat{X}_{\text{sig}}$  of the signal, obtained by a normalization of the output phase  $a_{\frac{\pi}{2}}^{\text{out}}$  as a displacement.  $\hat{X}_{\text{sig}}$  is the sum of the signal  $X_{\text{sig}}$  and extra noise terms,

$$\hat{X}_{\text{sig}} = \frac{1}{2\xi_a} a_{\frac{\pi}{2}}^{\text{out}} = X_{\text{sig}} + \frac{1}{2\xi_a} a_{\frac{\pi}{2}}^{\text{in}} + X_m. \quad (6)$$

The first noise term is related to the incident phase-noise  $a_{\frac{\pi}{2}}^{\text{in}}$  and corresponds to the measurement noise. The second term is the displacement  $X_m$  of mirror  $m$ . For a quantum-limited interferometer without control, it corresponds to the back-action noise due to radiation pressure of intracavity field  $a$ . It is deduced from the evolution of the velocity  $V_m = -i\Omega X_m$  which can be expressed from eq. (2) in terms of the incident intensity quadrature  $a_0^{\text{in}}$ ,

$$Z_m V_m = 2\hbar k_0 \alpha a_0 = \hbar \xi_a a_0^{\text{in}}, \quad (7)$$

where  $Z_m$  is the mechanical impedance of mirror  $m$ . At frequency relevant for gravitational-wave interferometers, a suspended mirror can be considered as a free mass with an impedance related to the mirror mass  $M_m$ ,

$$Z_m \simeq -i\Omega M_m. \quad (8)$$

Both noises in (6) are uncorrelated for an incident coherent state, the noise spectra for any quadrature  $a_\theta^{\text{in}}$  being given by [15],

$$\sigma_{a_\theta a_\theta}^{\text{in}} = 1, \quad \sigma_{a_\theta a_{\theta+\frac{\pi}{2}}}^{\text{in}} = 0, \quad (9)$$

where the spectrum  $\sigma_{a_\theta a_\theta}^{\text{in}}$  is the quantum average of the symmetrized product of quadratures,

$$\langle a_\theta^{\text{in}}[\Omega] \cdot a_{\theta'}^{\text{in}}[\Omega'] \rangle = 2\pi\delta(\Omega + \Omega') \sigma_{a_\theta a_{\theta'}}^{\text{in}}[\Omega]. \quad (10)$$

The sensitivity of the interferometer is described by the equivalent input noise  $\Sigma_{\text{sig}}$  equal to the spectrum of noises in the estimator  $\hat{X}_{\text{sig}}$ . One gets for a free interferometer,

$$\Sigma_{\text{sig}}^{\text{free}} = \frac{1}{4\xi_a^2} \left[ 1 + (\Omega_a^{\text{SQL}}/\Omega)^4 \right], \quad (11)$$

where  $\Omega_a^{\text{SQL}}$  is the frequency where contributions of both noises are equal,

$$\Omega_a^{\text{SQL}} = \sqrt{\frac{2\hbar\xi_a^2}{M_m}}. \quad (12)$$

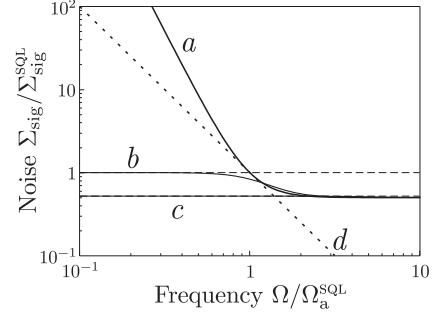


FIG. 2: Equivalent input noise  $\Sigma_{\text{sig}}$  in the interferometric measurement as a function of frequency  $\Omega$ . Curve  $a$ : free interferometer. Curves  $b$  and  $c$ : control with an optomechanical coupling  $\xi_b$  equal to  $\xi_a$  and  $5\xi_a$ , respectively; dashed curves correspond to an infinite gain and solid lines to the optimum gain. Curve  $d$ : standard quantum limit. Frequency is normalized to the SQL frequency  $\Omega_a^{\text{SQL}}$  and noise to  $\Sigma_{\text{sig}}^{\text{SQL}} = 1/2\xi_a^2$ .

As shown in curve  $a$  of Fig. 2, phase noise is dominant at high frequency with a flat frequency dependence, whereas radiation pressure is dominant at low frequency with a  $1/\Omega^4$  dependence. This behavior leads to the so-called standard quantum limit for a free interferometer with incident coherent light [3, 4, 5]. It corresponds to the minimum noise level reachable at a given frequency by varying the optomechanical coupling  $\xi_a$  (curve  $d$ ).

The sensor cavity measures the motion of mirror  $m$ . Since it is a resonant single-ended cavity, field  $b$  obeys equations similar to (2) and (3) except for the cavity length variation now equal to  $X_r - X_m$  ( $X_r$  is the displacement of the reference mirror  $r$ ). For this measurement we take advantage of the squeezing of light due to the self phase-modulation induced by radiation pressure [6]. Instead of measuring the phase quadrature  $b_{\frac{\pi}{2}}^{\text{out}}$ , we detect a properly chosen quadrature  $b_\theta^{\text{out}}$  of the reflected field. Equations for this quadrature and for mirror  $r$  are,

$$b_\theta^{\text{out}} = b_\theta^{\text{in}} + 2\xi_b \sin\theta (X_r - X_m), \quad (13)$$

$$Z_r V_r = \hbar \xi_b b_0^{\text{in}}, \quad (14)$$

where the optomechanical coupling  $\xi_b$  for cavity  $b$  is defined in the same way as  $\xi_a$  [eq. (5)]. Note that values of  $\xi_b$  as large as  $\xi_a$  are experimentally accessible with moderate incident power by using a high-finesse cavity [7]. In the following, mirror  $r$  is for simplicity assumed to be identical to mirror  $m$  so that  $Z_r = Z_m$ .

The sensor provides an estimator  $\hat{X}_m$  of the displacement of mirror  $m$ . The sensitivity for this measurement is limited by the phase noise of beam  $b$  and by the motion of mirror  $r$  due to radiation pressure,

$$\begin{aligned} \hat{X}_m &= -\frac{1}{2\xi_b \sin\theta} b_\theta^{\text{out}} \\ &= X_m - \frac{1}{2\xi_b} b_{\frac{\pi}{2}}^{\text{in}} - \left( \frac{\cot\theta}{2\xi_b} + \frac{i\hbar\xi_b}{\Omega Z_r} \right) b_0^{\text{in}}. \end{aligned} \quad (15)$$

Last term vanishes for a frequency-dependent value  $\theta^{\text{opt}}$  of quadrature angle  $\theta$  defined by,

$$\cot \theta^{\text{opt}} = (\Omega_b^{\text{SQL}}/\Omega)^2, \quad (16)$$

where  $\Omega_b^{\text{SQL}}$  is the SQL frequency for cavity  $b$  defined in a similar way as  $\Omega_a^{\text{SQL}}$  [eq. (12)]. The equivalent input noise for the sensor measurement is then only related to incident phase-noise and no longer depends on radiation-pressure effects on reference mirror  $r$ . Quadrature  $\theta^{\text{opt}}$  actually corresponds to the best strategy for the signal-to-noise ratio of the measurement [6]. At high frequency, radiation-pressure effects are negligible and the optimal quadrature is the phase  $b_{\frac{\pi}{2}}^{\text{out}}$  of the reflected field. As frequency decreases, radiation pressure becomes dominant and the optimal quadrature tends toward the intensity quadrature.

The result of the sensor measurement is fed back to mirror  $m$  via a force proportional to the estimator  $\hat{X}_m$ . The mirror motion in presence of feedback depends on radiation pressure of both cavities and on the feedback force,

$$Z_m V_m = \hbar \xi_a a_0^{\text{in}} - \hbar \xi_b b_0^{\text{in}} + i\Omega Z_{\text{fb}} \hat{X}_m, \quad (17)$$

where  $Z_{\text{fb}}$  is the transfer function of the feedback loop. For the optimum detection strategy [eq. (16)], the resulting motion of mirror  $m$  is,

$$(Z_m + Z_{\text{fb}}) V_m = \hbar \xi_a a_0^{\text{in}} - \hbar \xi_b b_0^{\text{in}} - \frac{i\Omega}{2\xi_b} Z_{\text{fb}} b_{\frac{\pi}{2}}^{\text{in}}. \quad (18)$$

The main effect of control is to change the response of mirror  $m$  to radiation-pressure by adding a feedback-induced impedance  $Z_{\text{fb}}$  to the free mechanical impedance  $Z_m$ . For a large feedback gain, the effective impedance is increased, then reducing mirror displacements. The control also contaminates mirror displacements by the noise in the sensor measurement [last term in eq. (18)]. Since this noise is only related to incident phase-noise of cavity  $b$ , the control can freeze the motion of mirror  $m$  in an absolute way, down to the limit associated to this noise. In contrast to a non-optimized measurement [7], the mirror locking is insensitive to the motion of reference mirror  $r$  induced by radiation-pressure fluctuations.

One gets from eq. (6) the estimator  $\hat{X}_{\text{sig}}$  for the interferometer in presence of feedback,

$$\begin{aligned} \hat{X}_{\text{sig}} &= X_{\text{sig}} + \frac{1}{2\xi_a} a_{\frac{\pi}{2}}^{\text{in}} + \frac{1}{2\xi_b} \frac{Z_{\text{fb}}}{Z_m + Z_{\text{fb}}} b_{\frac{\pi}{2}}^{\text{in}} \\ &\quad + \frac{i\hbar}{\Omega(Z_m + Z_{\text{fb}})} (\xi_a a_0^{\text{in}} - \xi_b b_0^{\text{in}}). \end{aligned} \quad (19)$$

Since all noises are uncorrelated the equivalent input noise is given by,

$$\Sigma_{\text{sig}} = \frac{1}{4\xi_a^2} + \frac{1}{4\xi_b^2} \left| \frac{Z_{\text{fb}}}{Z_m + Z_{\text{fb}}} \right|^2 + \frac{\hbar^2 (\xi_a^2 + \xi_b^2)}{\Omega^2 |Z_m + Z_{\text{fb}}|^2}. \quad (20)$$

For an infinite feedback gain, radiation-pressure noise is completely suppressed [last term in eq. (20)] and the equivalent input noise reduces to the sum  $1/4\xi_a^2 + 1/4\xi_b^2$  of phase noises of both cavities. The sensitivity no longer depends on frequency and tends to the phase noise of the interferometer alone as  $\xi_b$  increases (dashed curves in Fig. 2). The best sensitivity is obtained by optimizing the gain in eq. (20),

$$Z_{\text{fb}}^{\text{opt}} = Z_m (\Omega_{\text{fb}}/\Omega)^4, \quad (21)$$

$$\Sigma_{\text{sig}}^{\text{opt}} = \frac{1}{4\xi_a^2} + \frac{1}{4\xi_b^2 \left[ 1 + (\Omega/\Omega_{\text{fb}})^4 \right]}, \quad (22)$$

where frequency  $\Omega_{\text{fb}}$  is defined as,

$$\Omega_{\text{fb}}^2 = \Omega_b^{\text{SQL}} \sqrt{(\Omega_a^{\text{SQL}})^2 + (\Omega_b^{\text{SQL}})^2}. \quad (23)$$

For frequencies smaller than the cutoff frequency  $\Omega_{\text{fb}}$ , the resulting noise is similar to the one obtained for an infinite gain, whereas it reduces to the phase noise of the interferometer alone at high frequency. As shown in Fig. 2, one gets a clear increase of sensitivity at low frequency where back-action noise is completely suppressed and replaced by the phase noise of field  $b$ , without any loss at high frequency.

The optimized detection strategy [eq. (16)] is achieved by sending the field  $b^{\text{out}}$  in a detuned cavity and by detecting the reflected intensity (see Fig. 1). For a rigid and detuned cavity with a bandwidth comparable to frequency of interest, expression (2) of the intracavity field is modified as,

$$(\gamma + i\Delta - i\Omega\tau) c = \sqrt{2\gamma} c^{\text{in}} = \sqrt{2\gamma} b^{\text{out}}, \quad (24)$$

where  $\Delta$  is the detuning and  $\tau$  the round trip time of the cavity. According to the input-output relation (3) written for field  $c$ , the cavity simply induces a frequency-dependent rotation in phase space of quadratures, the output quadrature  $c_{\theta}^{\text{out}}[\Omega]$  being equivalent to the input quadrature  $b_{\theta-\phi_{\Omega}}^{\text{out}}[\Omega]$  with a rotation angle  $\phi_{\Omega}$  given by,

$$\cot \phi_{\Omega} = \frac{\gamma^2 - \Delta^2 + \Omega^2 \tau^2}{-2\gamma\Delta}. \quad (25)$$

Since the mean reflected field is rotated by an angle  $\phi_0$ , the measured intensity quadrature  $c_{\phi_0}^{\text{out}}[\Omega]$  corresponds to the quadrature  $b_{\phi_0-\phi_{\Omega}}^{\text{out}}[\Omega]$ . One gets the correct angle  $\phi_0 - \phi_{\Omega} = \theta^{\text{opt}}$  by choosing the cavity bandwidth  $\gamma/\tau$  and the detuning  $\Delta$  as,

$$\gamma/\tau = -\Delta/\tau = \Omega_b^{\text{SQL}}/\sqrt{2}. \quad (26)$$

Taking into account the global phase-shift experienced by the field in the detuned cavity, the input signal  $\hat{X}_m$  of the feedback loop [eq. (15)] is given by,

$$\hat{X}_m[\Omega] = -\frac{1}{2\xi_b} \frac{(\Omega_b^{\text{SQL}})^2 - \Omega^2 - \sqrt{2}i\Omega_b^{\text{SQL}}\Omega}{\Omega^2} c_{\phi_0}^{\text{out}}[\Omega]. \quad (27)$$

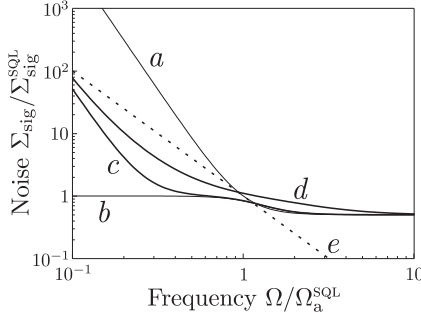


FIG. 3: Equivalent input noise  $\Sigma_{\text{sig}}$  in the interferometric measurement as a function of frequency  $\Omega$ . Curve *a*: free interferometer. Curve *b*: control without loss. Curves *c* and *d*: control with 1% loss in the sensor cavity ( $\eta_b = 0.01$ ), for the optimum feedback gain and for a single integrator transfer function, respectively. Curve *e*: standard quantum limit. Optomechanical couplings are equal ( $\xi_b = \xi_a$ ).

It corresponds to a causal filtering of the intensity fluctuations reflected by the detuned cavity.

We finally analyze the effects of optical losses on control performances. Losses in the interferometer can be accounted for by an additional damping coefficient  $\gamma_v$  for the cavity and by a coupling to a vacuum field  $v^{\text{in}}$  [15]. Eq. (2) is modified to,

$$(\gamma_a + \gamma_v)a = \sqrt{2\gamma_a}a^{\text{in}} + \sqrt{2\gamma_v}v^{\text{in}} + 2ik_0\alpha(X_m + X_{\text{sig}}). \quad (28)$$

Proportion of loss is defined by the coefficient  $\eta_a = \gamma_v / (\gamma_a + \gamma_v)$ . Losses in the sensor cavity are described by a similar equation with a coefficient  $\eta_b$ . For the same detection strategy as previously [eq. (16)], one can derive the optimum gain and the interferometer sensitivity,

$$Z_{\text{fb}}^{\text{opt}} = Z_m(1 - \eta_b) \frac{\Omega_{\text{fb}}^4}{\Omega^4 + \eta_b(\Omega_b^{\text{SQL}})^4}, \quad (29)$$

$$\Sigma_{\text{sig}}^{\text{opt}} = \frac{1}{4\xi_a^2(1 - \eta_a)} + \frac{1}{4\xi_b^2} \left[ \frac{1 - \eta_b}{1 + \eta_b(\Omega_b^{\text{SQL}}/\Omega)^4} + (\Omega/\Omega_{\text{fb}})^4 \right]^{-1} \quad (30)$$

which has to be compared to the sensitivity of the free interferometer in presence of loss, still given by eq. (11) with the phase-noise term 1 in the bracket replaced by  $1/(1 - \eta_a)$ . Losses in the interferometer ( $\eta_a > 0$ ,  $\eta_b = 0$ ) do not affect the control. One still has a complete suppression of back-action noise since only the phase noise due to field *a* is modified, in the same proportion for the free and controlled interferometers.

Cancellation of back-action noise is of course more sensitive to imperfections in the sensor measurement ( $\eta_b > 0$ ). Losses induce an additional cutoff frequency  $\sqrt[4]{\eta_b}\Omega_b^{\text{SQL}}$  both for the optimum gain and the sensitivity [compare eqs. (29), (30) to (21), (22)]. At lower fre-

quency the sensitivity can be approximated to,

$$\Sigma_{\text{sig}}^{\text{opt}} \simeq \eta_b \frac{1}{4\xi_b^2} (\Omega_b^{\text{SQL}}/\Omega)^4. \quad (31)$$

Cancellation is no longer perfect at low frequency. The sensitivity is contaminated by radiation-pressure effects in the sensor cavity, in proportion to the loss  $\eta_b$ . As shown in Fig. 3, however, one still has a reduction of back-action noise by a factor 100 for a 1% loss. Curve *d* finally shows the sensitivity obtained with a very simple implementation of the feedback loop which consists in a single integrator  $Z_{\text{fb}}/Z_m = 2\Omega_a^{\text{SQL}}/(-i\Omega)$ .

To conclude, a local control of mirrors with an optimized measurement strategy allows one to completely suppress back-action noise in interferometric measurement. This very efficient technique presents the advantage to decouple the constraints needed to manipulate quantum noise from the interferometer characteristics. As usual in quantum optics, losses must be avoided in the optomechanical sensor. Quantum locking is however insensitive to losses in the interferometer and does not imply any additional constraint to the interferometer design.

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